PROBLEM 1

Simplify the following expression using in part the direct calculation and in part the “nablaräkning”:

$$(a)\quad grad\left(fg\right)$$

$$(b)\quad div(f\,\overline{A})$$

$$(c)\quad rot(f\,\overline{A})$$

$$(d)\quad div\left(\overline{A}\times\overline{B}\right)$$

$$(e)\quad \left(\overline{A}\cdot\nabla\right)\overline{B}$$

Where:

$$f = r^3$$

$$g = 1/r^2$$

$$\overline{A} = (x^2, y^2, z^2)$$

$$\overline{B} = (z, y, x)$$

$$r = \sqrt{x^2+y^2+z^2}$$

PROBLEM 2

Use “nablaräkning” to calculate:

$$(a)\quad \left(\overline{b}\cdot\nabla\right)\left(\phi\overline{a}\right) =$$

$$(b)\quad \left(\overline{b}\cdot\nabla\right)\left(\overline{a}\times\overline{b}\right) =$$

PROBLEM 3

Write in suffix notation the following expressions:

$$(a)\quad \nabla\phi =$$

$$(b)\quad \nabla\cdot\overline{a} =$$

$$(c)\quad \nabla\times\overline{a} =$$

Use suffix notation (indexräkning) to prove the following expressions:

$$(d)\quad \nabla r = \hat{e}_r$$

$$(e)\quad \nabla\cdot\overline{r} = 3$$

$$(f)\quad \nabla\times\overline{r} = 0$$

Where:

$$\overline{r} = \left(x, y, z\right)$$

$$r = \sqrt{x^2+y^2+z^2}$$

PROBLEM 4

Use “nablaräkning” to show that:

$(a) \quad grad(\bar{a}\cdot\bar{r}) = \bar{a}$

$(b) \quad div(\bar{r}) = 3$

$(c) \quad div\left(\phi(r)\bar{r}\right) = 3\phi(r) + r\frac{d\phi}{dr}$

Where:

$\bar{r} = (x, y, z)$

$r = \sqrt{x^2 + y^2 + z^2}$

$\bar{a}, \bar{b}$ *are constant vectors*

PROBLEM 5

Calculate the integral: $\iiint_V \bar{A}\cdot\bar{B}dV$

Where $\bar{A}$ has a scalar potential: $grad\phi=\bar{A}$ and the boundary surface of V is an equipotential surface for ϕ. Moreover, $div\bar{B}=0$ in V.

PROBLEM 6

Calculate the flux integral: $\oiint_S (\bar{a}\times\bar{r})\times d\bar{S}$

Where $\bar{a}$ is a constant vector, $\bar{r}=(x,y,z)$ and S is a sphere with radius=1 centered in the point $\bar{b}$.